Exercise 34

Find equations of the tangent line and normal line to the given curve at the specified point.

$$y = \frac{2x}{x^2+1}, \quad (1,1)$$

Solution

Start by finding the slope of y at $x = 1$. Evaluate the derivative using the quotient rule.

$$\begin{aligned} y' &= \frac{d}{dx}\left(\frac{2x}{x^2+1}\right) \\ &= \frac{\left[\frac{d}{dx}(2x)\right](x^2+1) - \left[\frac{d}{dx}(x^2+1)\right](2x)}{(x^2+1)^2} \\ &= \frac{(2)(x^2+1) - (2x)(2x)}{(x^2+1)^2} \\ &= \frac{2-2x^2}{(x^2+1)^2} \end{aligned}$$

Evaluate it at $x = 1$.

$$y'(1) = 0$$

Therefore, the equation of the tangent line with slope 0 and the equation of the normal line with slope $-\infty$ that go through $(1,1)$ are respectively

$$y - 1 = 0(x-1) \quad \text{and} \quad x = 1.$$